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SHORTER COMMUNICATION

ON THE USE OF A BOUNDARY-LAYER MODEL FOR CORRELATING FILM COOLING DATA

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NOMENCLATURE

A, B,	constants, see text;
C,	concentration by mass;
C_p,	specific heat at constant pressure;
K,	concentration by volume;
M,	molecular weight of the gas;
T,	temperature;
h,	enthalpy;
m,	$\rho_c u_c/\rho_m u_m$;
Re_c, Re_x,	Reynolds numbers $\dfrac{\rho_c u_c s}{\mu_c}$, $\dfrac{\rho_m u_m x}{\mu_m}$;
s,	slot height;
u,	x-component of velocity;
x, y,	Cartesian coordinates;
δ,	boundary-layer thickness as measured by $y/\delta = 1$ when $u/u_m = 0.99$. (The boundary-layer concentration, temperature and velocity thicknesses are all assumed to be the same);
η',	film cooling effectiveness $(h_{aw} - h_{om})/(h_{oc} - h_{om})$;
μ,	coefficient of viscosity;
ρ,	density.

Subscripts

c,	coolant values at the slot exit;
m,	mainstream;
o,	total or stagnation;
aw,	adiabatic wall.

INTRODUCTION

As Spalding [1] has pointed out, a number of authors [2–4] have independently developed a very simple theory for the calculation of the adiabatic wall temperature produced by film cooling. In our previous note [2] this simple "boundary-layer model" was shown to correlate a wide range of data on film cooled adiabatic walls provided that the mass velocity parameter (m) was less than 1·5. For larger values of m the data considered in that note showed that the model was clearly inadequate in the range of distances from the slot currently of interest, $0 < x/s < 200$. More recently we have made further measurements using isothermal foreign gas injection to cover a wider range of jet-to-freestream density ratios. The analogy between heat- and mass-transfer has enabled the results to be related to the film cooling effectiveness.

The extension of the boundary-layer model to predict the wall concentration and a re-examination of the importance of m are the subjects of this communication.

ANALYSIS

The assumptions here are:

(a) the flow is boundary-layer-like;

(b) the velocity, temperature and concentration boundary layers have the same thickness, δ, which is given by

$$\delta = A_1\, x\, Re_x^{-\frac{1}{5}}; \tag{1}$$

(c) the mass-velocity profile may be expressed as a power law with $n = \frac{1}{7}$, i.e.

$$\frac{\rho u}{\rho_m u_m} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}; \tag{2}$$

(d) the concentration profile is given by

$$\frac{C}{C_{aw}} = \exp\left[-A_2(y/\delta)^2\right] \text{ for } 0 \leqslant y \leqslant \delta \tag{3}$$

and by $C = 0$ for $y > \delta$.

The conservation of mass through the boundary layer demands that

$$\rho_c u_c s = \int_0^\delta \rho u C \, dy \tag{4}$$

or

$$\frac{\rho_c u_c s}{\rho_m u_m \delta} = C_{aw} \int_0^1 \frac{\rho u}{\rho_m u_m} \cdot \frac{C}{C_{aw}} \cdot d(y/\delta) = C_{aw} \cdot A_3 \tag{5}$$

where A_3 is the numerical value of the above integral.

Thus

$$C_{aw} = ms/A_3\delta \tag{6}$$

which may be rearranged using equation (1) to give

$$C_{aw} = B_1\left(\frac{x}{ms}\right)^{-0.8}\left(Re_c\frac{\mu_c}{\mu_m}\right)^{0.2}. \tag{7}$$

The left-hand side of equation (7) may be written as

$$\{C_{aw} - C_m\}/\{C_c - C_m\}$$

since $C_c = 1$ and C_m is zero. Equation (7), thus modified, may be compared with the corresponding relation for the film cooling effectiveness, as derived for example in reference [2], namely

$$\eta' = \frac{h_{aw} - h_{om}}{h_{oc} - h_{om}} = B_2\left(\frac{x}{ms}\right)^{-0.8}\left(Re_c\frac{\mu_c}{\mu_m}\right)^{0.2}. \tag{8}$$

The wall concentration by volume $K_{aw}(x)$ is related to C_{aw} by

$$C_{aw} = \left[1 + \frac{M_m}{M_c}\left\{\frac{K_c - K_{aw}}{K_{aw} - K_m}\right\}\right]^{-1} \tag{9}$$

where $K_c = 1$ and $K_m = 0$.

The corresponding expression for η' in terms of total temperature is [2],

$$\eta' = \left[1 + \frac{C_{p_c}}{C_{p_m}}\left\{\frac{T_{oc} - T_{aw}}{T_{aw} - T_{om}}\right\}\right]^{-1}. \tag{10}$$

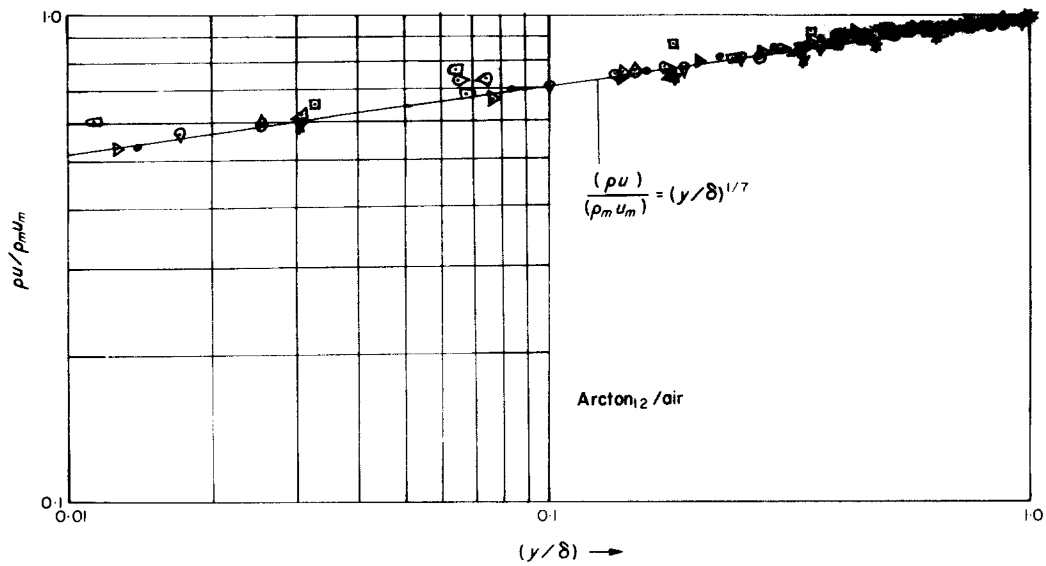

FIG. 1. Correlation of normalized mass velocity profiles in the fully developed turbulent boundary-layer region.

Key Symbols	x/s	u_c/u_m
□	1008	1·45
⊙	124·7	0·80
◺	207	
◿	670	
▷	1008	
▢	124·7	0·485
▢	207	
△	670	
▽	1008	
⬡	53·5	0·245
▷	124·7	
◁	207	
✡	670	
●	1008	

$s = \frac{1}{16}$ in for all tests.

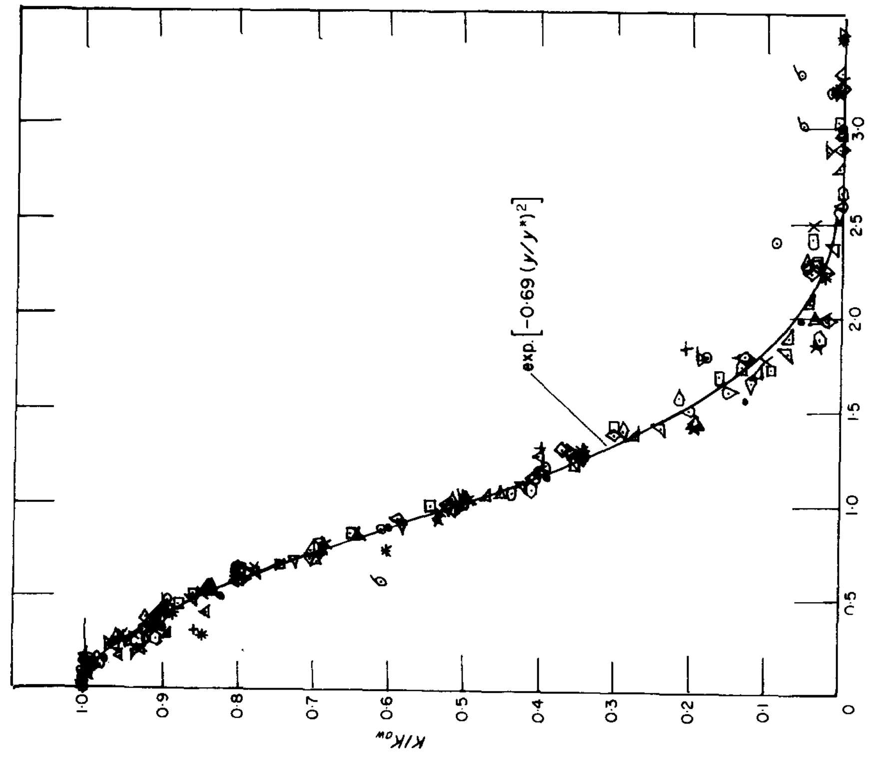

FIG. 2. Correlation of normalized concentration profiles across the boundary layer with Arcton$_{12}$ injection.

Key

Symbols	x/s	u_c/u_m
σ	19·0	0·245
⊙	53·5	
◊	124·7	
▽	207·0	
✦	670	
●	1008	
+	19·0	0·485
×	53·5	
⊡	124·7	
⊡	207	
◁	670	
▷	1008	
◁	19·0	0·80
▷	53·5	
⊙	124·7	
◿	207	
◁	670	
△	1008	
✱	19·0	1·45
▽	53·5	
◁	124·7	
◇	207	
◇	670	
⊡	1008	

$s = \frac{1}{16}$ in for all tests.

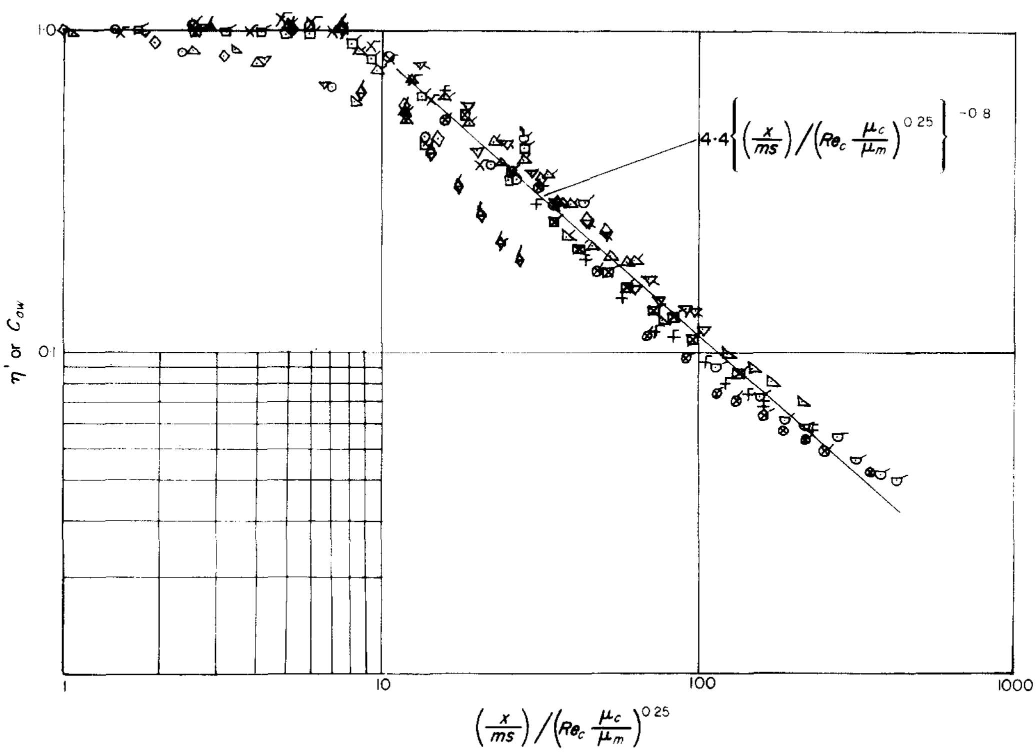

FIG. 3. The values of C_{aw} for Arcton_{12}/air and the data of Hatch and Papell [6] for T_{aw} using helium/air ($0 \leqslant u_c/u_m \leqslant 1{\cdot}5$) are compared with the boundary-layer model of the flow.

Key	Symbols	u_c/u_m	s''	m	Gas
	⊙	1·45	$\frac{1}{16}$	6·1	Arcton_{12}/air (reference 5)
	◇	1·0	$\frac{1}{16}$	4·2	Arcton_{12}/air (reference 5)
	△	0·8	$\frac{1}{16}$	3·35	Arcton_{12}/air (reference 5)
	▽	0·485	$\frac{1}{16}$	2·04	Arcton_{12}/air (reference 5)
	◺	0·245	$\frac{1}{16}$	1·05	Arcton_{12}/air (reference 5)
	△̸	1·484	$\frac{1}{8}$	0·507	Helium/air (reference 6)
	□̸	1·217	$\frac{1}{2}$	0·447	Helium/air (reference 6)
	×̄	1·108	$\frac{1}{2}$	0·290	Helium/air (reference 6)
	▽̸	1·097	$\frac{1}{8}$	0·361	Helium/air (reference 6)
	⊠	0·859	$\frac{1}{8}$	0·272	Helium/air (reference 6)
	+̄	0·591	$\frac{1}{8}$	0·181	Helium/air (reference 6)
	◇̸	0·552	$\frac{1}{2}$	0·188	Helium/air (reference 6)
	⊗	0·433	$\frac{1}{8}$	0·125	Helium/air (reference 6)
	⊖̸	0·327	$\frac{1}{8}$	0·084	Helium/air (reference 6)

RESULTS

A selection of some experimental results* taken from reference [5] are shown in Figs. 1–3. The data are for a wall jet of Arcton_{12} (molecular weight 120·9) and an air mainstream.

Figure 1 is a logarithmic plot of the mass-velocity (ρu) profiles measured at five stations along the plate. The $\frac{1}{7}$th power law profile is shown for comparison. All but six of the 96 data points lie within ± 10 per cent of the assumed power law distribution. The data correlation improves as u_c/u_m decreases and x/s increases. This trend is as expected since the real flow must asymptote to a conventional boundary-layer flow far downstream from the jet exit slot.

Figure 2 compares the measured concentration profiles† with a Gaussian distribution. The curve does not fit some of the data taken close to the slot ($x/s = 19$), but 95 per cent of the remaining data deviate from the curve by an amount which is less than the accuracy of the measurements. The experimental results suggest that the concentration profiles are similar and accurately described by a Gaussian distribution provided $x/s \geqslant 50$.

Figure 3 shows two sets of independent experimental results covering a wide range of density ratios, slot heights and Reynolds numbers. Arcton_{12} wall concentration data from reference [5] are compared with some adiabatic wall temperature measurements reported by Hatch and Papell [6] who used helium as the film coolant. Though the data selected‡ from these references cover the range $0{\cdot}084 \leqslant m \leqslant 6{\cdot}1$ all the velocity ratios u_c/u_m are less than 1·5. Considering the simplicity of the analytical model the ability of equations (7) and (8) to *correlate* the data is very satisfactory. It must be emphasized therefore that it is not the value of m, as previously implied, that limits the utility of the boundary-layer model but the value of the velocity ratio u_c/u_m. Provided $u_c/u_m \leqslant 1{\cdot}5$ the boundary-layer model correlates the data for all m. For $u_c/u_m > 1{\cdot}5$ the correlation is unsatisfactory, [2]. This is not surprising since a boundary-layer model cannot describe jet-like flows except at extreme distances from the slot.

For the purposes of prediction Fig. 3 is less satisfactory. A proportionality constant, B, of 4·4 gives the best fit to the limited data but nothing can (or should) disguise the fact that at a given value of the abscissa the ordinate can vary by as much as 100 per cent.

* These results are taken from a more general investigation into the turbulent mixing of foreign gas jets with a moving airstream [5] which will be more fully reported later.

† The measured concentration by volume (K) is shown in Fig. 2, rather than the concentration by mass (C). $C(y)$ has been derived from the measurements, using equation (9), and plotted. The resultant distribution confirms that equation (3) is an adequate representation.

‡ The data for $m \leqslant 0{\cdot}08$ showed great scatter (see reference [2]) and have not been plotted. It is felt that errors in measuring the very low helium velocity may have been the primary cause of the scatter.

CONCLUSIONS

(1) The boundary-layer model of the flow correlates the data presented here covering the range $0 < m \leqslant 6{\cdot}1$ with $u_c/u_m \leqslant 1{\cdot}5$. Reference 2 showed that for larger velocity ratios the boundary-layer model does not correlate the data. The inference is that the boundary-layer model of the flow will correlate experimental measurements of film cooling effectiveness even when there are large density differences between the coolant and mainstream gases provided that $u_c/u_m \leqslant 1{\cdot}5$.

(2) With the above proviso the boundary-layer model is also useful in studying the isothermal mixing of a wall-jet exhausting into a moving stream of foreign gas.

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